WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMADSE05T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) What is an isotone from a poset P into a poset Q? Show that each of the operations of join and meet in a lattice L induces an isotone from L into itself.

(b) Define direct product of two ordered sets P and Q. Prove that the direct product LM of two lattices L and M is a lattice.

(c) Let L be a distributive lattice and $c, x, y \in L$. If in L, $c \wedge x = c \wedge y$ and $c \vee x = c \vee y$, then show that $x = y$.

(d) Karnaugh map for a Boolean polynomial $f(x, y, z)$ in three Boolean variables x, y, z is given below:

$z \backslash xy$	00	01	11	10
0	0	1	1	1
1	0	0	0	1

Determine $f(x, y, z)$ from the above map and then express it in DNF in the variables x, y, z.

(e) Simplify the regular expression

$$01^*1 + 11^*01^*1 + (0 + 01^*1)1^*1$$

on the alphabet $\Sigma = \{0, 1\}$.

(f) State the Pumping Lemma for regular languages.

(g) Define a Turing machine.

(h) Construct a context-free grammar that generates the following language:

$$\{\omega c \omega^R : \omega \in \{0, 1\}^*\}$$

(i) Let $\Sigma = \{a, b\}$. Write regular expression for the following set:

All strings in Σ^* with exactly one occurrence of the substring *aaa*

2. (a) Consider the ordered set $(\mathbb{N}, \leq)$, where for any $a, b \in \mathbb{N}$, $a \leq b$ if and only if $a \mid b$. Show that $(\mathbb{N}, \leq)$ is a lattice. Is this lattice complete? Justify your answer. 2

(b) Let P be a finite ordered set. Then, for any $x, y \in P$, prove that $x < y$ if and only if there exists a finite sequence of covering relation $x = x_0 \prec x_1 \prec \ldots \prec x_n = y$. 2

(c) Let P and Q be two finite ordered sets and $\phi : P \to Q$ be a bijective mapping. Then, prove that the following assertions are equivalent: 4

(i) ϕ is an order isomorphism

(ii) $x \prec y$ in $P \Leftrightarrow \phi(x) \prec \phi(y)$ in Q.

3. (a) Let $f : B \to C$, where B and C are Boolean algebras. 5

(i) Assume f to be a lattice homomorphism. Then prove the following to be equivalent:

- $f(0) = 0$ and $f(1) = 1$
- $f(a') = (f(a))'$, $\forall a \in B$.

(ii) Also prove that, if f preserves$'$, then (f preserves $\vee$ if and only if f preserves $\wedge$).

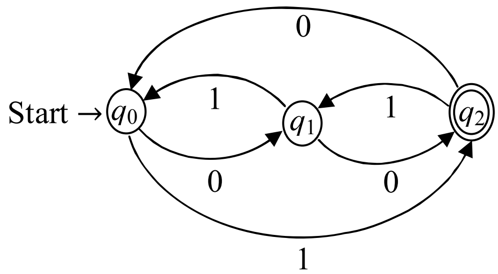
(b) Let L and K be lattices and $f : L \to K$ a map. Prove the following to be equivalent: 3

(i) f is order preserving

(ii) $(\forall a, b \in L)\ f(a \vee b) \geq f(a) \vee f(b)$

(iii) $(\forall a, b \in L)\ f(a \wedge b) \leq f(a) \wedge f(b)$.

4. (a) Convert the following DFA to a regular expression. 4



(b) Let $\Sigma = \{a, b\}$. Let L be a language over Σ, consisting of strings of length at least 2, where the first letter is the same as the last letter, and the second letter is the same as the second to last letter. For example, $a \notin L$, $b \notin L$, $aa \in L$, $aaa \in L$, $aba \in L$, $bbaabba \notin L$. Design a DFA that accepts L. 4

5. (a) Kleene closure of a regular language A, is defined as $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0$ and each $x_i \in A\}$. Prove that Kleene closure of every regular language is regular. 4

(b) Prove that every non-deterministic finite automaton can be converted to an equivalent one that has a single accept state. 4

6. (a) Draw the state diagram of a Pushdown Automata realizing $\{0^n 1^n \mid n \geq 0\}$. 3

(b) Prove that any context-free language is generated by a context-free grammar in Chomsky normal form. 5

7. (a) Prove that the normal subgroups of a group form a modular lattice, under set inclusion. 3

(b) How many minimal Boolean polynomials are there, in n Boolean variables? When is a Boolean polynomial said to be in Disjunctive Normal Form (DNF)? 2

(c) Convert the Boolean polynomial 3

$$f(x, y, z, t) = xyzt + x'y'zt + xyz't + xy'z't + xyz't' + xy'z't'$$

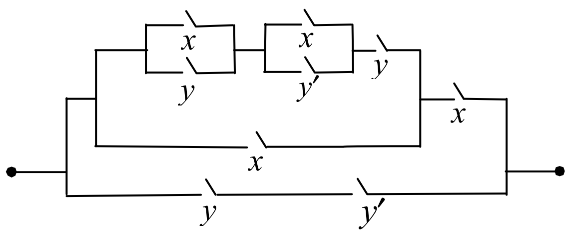
into its minimal form.

8. (a) Truth table for a Boolean polynomial $f(x, y, z)$ in three variables x, y, z is given below: 5

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

From this table, form Karnaugh map for the polynomial $f(x, y, z)$, taking x along the row and yz along the column of the map. From the Karnaugh map determine $f(x, y, z)$. Then, construct the logic circuit (of the logic-gates) for $f(x, y, z)$.

(b) Find the Boolean polynomial which represents the following switching circuit: 2+1



Hence, draw an equivalent circuit as simple as possible.

9. (a) Let $G = (V, \Sigma, S, P)$ be a context-free grammar (CFG), where $V = \{S, A, B\}$ is the set of non-terminals, $\Sigma = \{a, b, c\}$ is the set of terminals, S is the start symbol and P is the set of productions $S \to ABa,\ A \to aab,\ B \to Ac$. Transform this grammar G into a CFG in Chomsky Normal Form. 6

(b) Define ID of a Non-deterministic Pushdown Automaton (NPDA). 2

10.(a) Construct the NPDA which accepts the context-free language L on the alphabet $\Sigma = \{a, b, c\}$, generated by the CFG, G with variables A, B, C; the start variable S and productions 4

$$S \to aA$$
$$A \to aABC|bB|a$$
$$B \to b$$
$$C \to c$$

(b) Describe how can a Turing Machine be made as a unary to binary converter. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——