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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2019

PHSACOR05T-PHYSICS (CC5)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and *two* other questions from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) For the function $f(x)=|x|, -1<x<1$, the Fourier series expansion for $-1<x<1$ will not contain any term of the form $\sin nx$, n being an integer. Justify this statement.

(b) Write Dirichlet Conditions in connection with Fourier series expansion.

(c) Show that for Laguerre equation $xy''+(1-x)y'+ny=0$ there is an essential singularity at infinity.

(d) Derive the recurrence relation for the gamma functions:

$$\Gamma(n+1)=n\Gamma(n).$$

(e) Using the generating function of Bessel function, given by

$$G(x,t)=e^{\frac{x}{2}(t-1/t)}.$$

Prove that $J_{n-1}(x)+J_{n+1}(x)=\frac{2n}{x}J_n(x)$.

(f) The error function $\text{erf}(x)$ is defined by

$$\text{erf}(x)=A\int_0^x e^{-z^2}dz.$$

Find the normalization of $\text{erf}(x)$.

(g) Consider the differential equation,

$$x(x-1)y''+3xy'+y=0.$$

Identify its singular points and classify the singularities.

(h) What will you get if you calculate the Poisson bracket $\{p, p^2+x^2\}$, where x is the position coordinate and p is the corresponding generalized momentum.

(i) Show that the general solution of the wave equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},$$

is of the form $u(x,t)=f(x-ct)+g(x+ct)$.

(j) Set up the equation of motion of a one-dimensional simple harmonic oscillator using Lagrange's equation.

(k) Find the Legendre Transform of x^2.

(l) Plot schematically the functions $J_0(x)$ and $J_1(x)$ on the same graph.

(m) Three mass points are sitting on the vertices of a triangle of fixed arm-lengths. Calculate the degrees of freedom of the system.

(n) Hermite polynomials obey the following recursion relation.

$$H_{n+1}(x)=2xH_n(x)-2nH_{n-1}(x)$$

Given, $H_0(x)=1$ and $H_1(x)=2x$.

Find $H_4(x)$.

2. (a) Given, $f(x)=\begin{cases}-1, & -\pi<x<0\\ +1, & 0<x<\pi\end{cases}$ 3

Expand $f(x)$ in an appropriate Fourier series of period 2π.

(b) Given, $\vec{r}_{12}=\vec{r}_2-\vec{r}_1$, where $\vec{r}_1$ and $\vec{r}_2$ are respective position vectors of the points P_1 and P_2. Expand $\frac{1}{|\vec{r}_{12}|}$ in terms of Legendre polynomials. How do you interpret this result? 3+1

(c) Define Hamiltonian as a Legendre transform of the Lagrangian. Hence derive Hamilton's equations of motion. 1+2

3. (a) Let $f(x)$ have a Fourier series expansion, 3

$$f(x)=\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx,\ (a_n \text{ and } b_n \text{ are real constants})$$

Prove that, $\left\langle f^2(x)\right\rangle \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi}f^2(x)dx=\sum_{n=1}^{\infty}\frac{a_n^2+b_n^2}{2}$.

(b) Show that for the following equation, 2+2

$$xy''+(1-x)y'+4y=0,$$

about $x=0$, the only possible solution of the indicial equation is 0.

Find the recursion relations among the coefficients appearing in the Frobenius series.

(c) Show that for any dynamical variable u, 2+1

$$\frac{du}{dt}=\frac{\partial u}{\partial t}+\{u, H\},\ \text{where } \{\}\ \text{stands for Poisson bracket.}$$

Hence prove that the Hamiltonian itself is a constant of motion when it does not explicitly depend on time.

4. (a) Write down the Lagrangian of a particle moving under the influence of a force $\vec{F} = -\frac{k}{r^2}\hat{r}$. 3

(b) Considering the solution of the Bessel's equation, 3

$$x^2y'' + xy' + (x^2 - n^2)y = 0,$$

in the form of $y(x) = \sum_{p=0}^{\infty} a_n x^{p+m}$, show that the roots of the indicial equation are $m = \pm n$.

(c) Define beta function $B(m, n)$ and gamma function $\Gamma(n)$. 2+2

Show that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

5. (a) Solve $\nabla^2\phi = 0$ using separation of variables in Cartesian coordinates. Hence find $\phi(x, y, z)$ inside a cube of side L on each face of which $\phi(x, y, z) = \phi_0$, a constant. 3+1

(b) Using Euler-Lagrange equation show that the shortest path on a plane connecting two points is a straight line. 3

(c) Starting from the Rodriguez formula: 3

$$P_l(x) = \frac{1}{2^l\, l!} \cdot \frac{d^l}{dx^l}(x^2 - 1)^l,$$

for Legendre polynomial $P_l(x)$ of degree l, show that

$$\int_{-1}^{1} P_l(x)\, P_l(x) dx = \frac{2}{2l+1}.$$

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