



WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

PHSADSE04T-PHYSICS (DSE3/4)

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

Answer Question No.1 and any *two* questions from the rest

1. Answer any *fifteen* questions from the following: 2×15 = 30
- For a Poisson distribution; $P(x=0) = P(x=1)$ find the $P(x>0)$.
 - What do you mean by proper subset? Explain with an example.
 - Distinguish between Onto and Into mapping.
 - If N is the set of all positive integers, then show that division and subtraction are not binary operations in N .
 - When can a binary relation be called an equivalence relation?
 - For what value of a will be the function $f(x) = ax$; $x = 1, 2, 3, \dots, n$ be the probability mass function of a discrete random variable x ?
 - Show that the set of matrices $A_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, where α is real forms a group under multiplication.
 - Prove that if event A and B are independent, \bar{A} and \bar{B} are also independent.
 - Show that the identity of a subgroup of a group is the same as that of the group.
 - Show that the intersection of any two normal subgroups of a groups is a normal subgroup.
 - If (R, \cdot) be a multiplicative group and $x \in R$, then find homomorphisms and their kernels in the mapping $x \rightarrow |x|$.
 - What do you mean by faithful representation? Give an example.
 - If in an experiment with random outcome the number of outcomes favouring an event A is less than the number of outcomes favouring event B in a large number of trials N , show that $P(A) < P(B)$. The symbols have their usual meaning.
 - A dice is rolled six times. Find the probability that each of the six faces appears only once.
 - If $f(x) = e^{-x}$, $0 \leq x \leq \infty$ show that it is a probability density function and find its mean.
 - Write down the limitations of separation of variable technique of solving partial differential equation.

(q) Explain with example, what are homogeneous and non-homogeneous partial differential equations.

(r) Show that if there exists a relation among the three variables x, y, z like

$$f(x, y, z) = 0, \text{ then show that } \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1.$$

(s) For what value of x and y the equation

$$(y+1)\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = x+y$$

is parabolic?

(t) Write down the Laplace's equation in spherical polar coordinates.

2. (a) Define order of an element of a group. Prove that the order of an element a of a group is the same as that of its inverse a^{-1} . 3
- (b) Show that the group formed by the set $(1, \omega, \omega^2)$, ω being cube root of unity, is a cyclic group of order 3 with respect to multiplication. 2
- (c) Write down orthogonality theorem for the Irreducible representation of a group and prove it. 2+3
3. (a) Find the regular permutation group isomorphic to the group $G = (a, b, c, d)$ with the composition table. 3
- (b) If a matrix commutes with all the matrices of an irreducible representation, then show that it is a multiple of unit matrix. 4
- (c) For any two events A and B , the probability that either A or B or both occur is given by $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. 3
4. (a) Consider the Gaussian distribution $\rho(x) = Ae^{-\lambda(x-a)^2}$, where A, a, λ are constants. Determine σ^2 . 4
- (b) If A and B are two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$. Are A and B independent? 3
- (c) If X denotes a random variable having binomial distribution with mean 6 and variance 3. Obtain $P(X \geq 1)$. 3
5. (a) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ where $u = 0$ for $t = \infty$ and for $x = 0$ or 1 . 4
- (b) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2} = 0$ for $0 < x < \pi$, $0 < y < \pi$ subject to the conditions 6
 $u(0, y) = u(\pi, y) = u(x, \pi) = 0$ and $u(x, 0) = \sin^2 x$.

N.B. : Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

—x—