WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 6th Semester Examination, 2021

MTMGDSE03T-MATHEMATICS (DSE2)

NUMERICAL METHODS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

   (a) Construct a linear interpolation for $f(x)$ with $f(1)=3$ and $f(2)=-5$.

   (b) Compute $\int_0^4 2x^3\,dx$, by Simpson's $\frac{1}{3}$ formula and comment on the result.

   (c) Derive an iteration formula for computing $\sqrt[3]{a}$, using Newton Raphson method.

   (d) What is the condition of convergency of Gauss-Jacobi iteration to solve the system of n linear equations? Is this condition both necessary and sufficient?

   (e) Show that the equation $x^2+\ln x=0$ has exactly one root in the interval $\left[\frac{1}{3}, 1\right]$.

   (f) If 0.667 be an approximate value of $\frac{2}{3}$, find the percentage error.

   (g) What do you mean by Numerical Differentiation?

   (h) Show that $\Delta^2 \cos 2x = 4\cos 2x$ where interval of differencing is $\frac{\pi}{2}$.

   (i) Define the terms absolute and relative errors.

2. Explain the Newton-Raphson method for computing a simple real root of an equation $f(x)=0$. When does the method fail? Can we apply this method to the equation $x^2-x+\frac{1}{4}=0$? Justify your answer. 4+1+1+2

3. (a) In order to find the root of $x^3-x-1=0$, near $x=1$ which of the following iteration functions give convergent sequences: 2+2

   (i) $x=\frac{x+1}{x^2}$ (ii) $x=\sqrt{\frac{x+1}{x}}$

   (b) Apply the method of bisection to find a real root up to two significant digits of the equation $x^3-3x-5=0$. 4

4. (a) Use Lagrange's interpolation to find the value of $f(x)$ for $x = 0.4$ using the table. 5

x	0.3	0.5	0.6
$f(x)$	0.61	0.69	0.72

(b) Find $\Delta^4 f(x)$, where $f(x) = (3x+2)(x-2)(x+1)(5x-1)$ and the interval of differencing is unity. 3

5. What is interpolation? Deduce Newton's forward difference interpolation formula without error term. 8

6. (a) Given the following table: 2+3

x	0	5	10	15	20
$f(x)$	1.0	1.6	3.8	8.2	15.4

Construct the difference table and compute $f'(20)$.

(b) If $f(x) = ax$, show that $(E + E^{-1}) f(x) = 2 f(x)$. 3

7. (a) Calculate $\int_1^2 (x + \frac{1}{x})\, dx$ up to four significant figures by Simpson's $\frac{1}{3}$ rule taking 4 intervals. 4

(b) Obtain trapezoidal rule for numerical integration without the error term. 4

8. Solve the system of equations by LU decomposition method: 8

$$3x + 4y + 2z = 15 \ , \ 5x + 2y + z = 18 \ , \ 2x + 3y + 2z = 10$$

9. Deduce Lagrange's interpolation formula and also prove that Lagrangian functions are invariant under linear transformation. 8

10.(a) For any positive integer k, show that 4

$$\nabla^k y_n = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{n-i}$$

∇ being the backward difference operator.

(b) What do you mean by ' round off ' errors in numerical data? Show how these errors are propagated in a difference table. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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