WEST BENGAL STATE UNIVERSITY
B.Sc. Honours 6th Semester Examination, 2021

MTMADSE04T-MATHEMATICS (DSE3/4)

THEORY OF EQUATIONS

Time Allotted: 2 Hours   Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: 2×5 = 10

    (a) If $x^4 + px^2 + qx + r$ can be expressed in the form $(x-a)^3(x-b)$, show that $8p^3 + 27q^2 = 0$.

    (b) Find the quotient and remainder when $x^3 + 5x^2 + 1$ is divided by $x + 3$.

    (c) If α, β, γ be the roots of $x^3 + qx + r = 0$, prove that $\sum \dfrac{1}{\beta + \gamma - \alpha} = \dfrac{q}{2r}$.

    (d) The equation $x^n - nx + n - 1 = 0 \, (n > 1)$ is satisfied by $x = 1$. What is the multiplicity of this root?

    (e) Use Strum's theorem separate the roots of the equation $3x^4 - 6x^2 - 8x - 3 = 0$.

    (f) Verify whether the following functions are symmetric or not:

    (i) $f(x, y, z) = x^2 y^2 + y^2 z^2 + z^2 x^2$

    (ii) $f(x, y, z) = xy + yz$.

    (g) Multiply the roots of the equation $x^4 + \dfrac{1}{2}x^3 + \dfrac{1}{4}x + \dfrac{5}{12} = 0$ by a suitable constant so that the fractional co-efficients of the equation may be removed.

    (h) Define reciprocal equation.

    (i) Find the number of special roots of the equation $x^n - 1 = 0$, when n is a prime and $n = p^\alpha$, where p is a prime and α is a positive integer > 1.

2. (a) If $f(x)$ be a polynomial then prove that $(x - \alpha)$ is a factor of $f(x)$ if and only if $f(\alpha) = 0$.   2+2

    (b) Show that $x^{20} + x^{15} + x^{10} + x^5$ is divisible by $x^2 + 1$.   4

3. (a) If α be a special root of the equation $x^n - 1 = 0$, then prove that $\dfrac{1}{\alpha}$ is also a special root of it.   4

    (b) If α be a special root of the equation $x^{12} - 1 = 0$, prove that $(\alpha + \alpha^{11})(\alpha^5 + \alpha^7) = -3$.   4

4. (a) Find the equation whose roots are the roots of the equation $x^3 + 3x^2 - 8x + 1 = 0$ 2+2
(i) each diminished by 4, (ii) increased by 1.

  (b) Find the relation among the coefficients of the equation   4
$a_0 x^3 + 3a_1 x^2 + 3a_2 x + a_3 = 0$ so that the second term and the third term may be removed by the transformation $x = y + h$.

5. (a) Find an upper limit of the real roots of the equation $x^4 - 2x^3 + 3x^2 - 2x + 2 = 0$.   4

  (b) Calculate Sturm's functions and locate the position of the real root of the equation   4
$x^4 - x^2 - 2x - 5 = 0$.

6. (a) If an equation with rational coefficients has a surd root of $\alpha + \sqrt{\beta}$, where α, β 4
are rational and β is not a perfect square, then show that it has the conjugate root $\alpha - \sqrt{\beta}$.

  (b) Determine r so that one root of the equation $x^3 - rx^2 + rx - 4 = 0$ shall be   4
reciprocal of another and find all the roots.

7. (a) State Newton's theorem. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots of the equation   2+2
$x^4 + p_2 x^2 + p_3 x + p_4 = 0$. Find the value of $\sum \alpha^3$ by Newton's theorem.

  (b) Solve $3x^3 - 26x^2 + 52x - 24 = 0$ given that the roots are in geometric progression.   4

8. (a) Reduce the biquadratic $2x^4 - 4x^3 + 3x^2 + 2x + 3 = 0$ into standard form.   4

  (b) If α, β, γ be the roots of $x^3 + px + q = 0$, prove that $6S_5 = 5S_2 S_3$, where   4
$S_r = \sum \alpha^r$.

9. (a) Find the number and position of the real roots of the equation $x^5 - 5x + 1 = 0$.   6

  (b) Show that the equation $(x - a)^3 + (x - b)^3 + (x - c)^3 + (x - d)^3 = 0$, where a, b, c, d 2
are not all equal, has only one real root.

10.(a) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find an equation   6
whose roots are $\dfrac{1}{\alpha} + \dfrac{1}{\beta} - \dfrac{1}{\gamma}$, $\dfrac{1}{\beta} + \dfrac{1}{\gamma} - \dfrac{1}{\alpha}$, $\dfrac{1}{\alpha} + \dfrac{1}{\gamma} - \dfrac{1}{\beta}$.

  (b) If $\alpha, \beta, \gamma, \delta$ be the roots of $x^4 - 3x^3 + 4x^2 - 5x + 6 = 0$, find the value of   2
$(\alpha^2 + 3)(\beta^2 + 3)(\gamma^2 + 3)(\delta^2 + 3)$.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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