WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

PHSACOR14T-PHYSICS (CC14)

STATISTICAL MECHANICS

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) Explain the statistical idea of entropy.

(b) What is meant by the term 'equal a priori probability'?

(c) Draw a phase space trajectory of a simple harmonic oscillator of energy E.

(d) A classical particle is free to move in a cube of side l. If its energy $\leq E$ find the volume of the phase space available to it.

(e) What is ergodic hypothesis?

(f) What do you mean by 'ultraviolet catastrophe'?

(g) Find differences among microcanonical, canonical and grand canonical ensembles.

(h) What is the most probable kinetic energy $\tilde{\varepsilon}$ corresponding to Maxwellian velocity distribution?

(i) State Pauli's exclusion principle.

(j) What are distinguishable and indistinguishable particles?

(k) What is Bose-Einstein condensation?

(l) Show that the volume element

$$d\tau = \prod_{i=1}^{3N} (dq_i dp_i)$$

of the phase space remains invariant under a canonical transformation.

(m) State and explain Wien's displacement law.

(n) Prove that total pressure of diffused radiation is $P_{\text{rad}} = \frac{1}{3}u$, u being the energy density of radiation.

(o) From the knowledge of partition function Z, write an expression for entropy S in ideal Fermi gas.

Answer any *two* questions from the following 10×2 = 20

2. (a) Distinguish between microstates and macrostates. 2

   (b) Two dices are rolled simultaneously. Write the number of microstates and number of macrostates. 2

   (c) What is meant by a stationary ensemble? Give one example of a stationary ensemble. 2

   (d) Write all possible microstates of two quantum harmonic oscillators having total energy 4ε, ε being the spacing between the energy levels. Neglect the zero point energy. 2

   (e) State the principle of equipartition of energy. 2

3. (a) Show that average energy, $\langle E \rangle = -\dfrac{\partial \ln Z}{\partial \beta}$ where $Z = \sum_r e^{-\beta E_r}$ is the partition function. 2

   (b) Consider a system consisting of N independent harmonic oscillators, whose Hamiltonian is given by,

$$H(p,q) = \frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 q_i^2, \ (i = 1, 2, \ldots, N).$$

   (i) Calculate the partition function for the system using canonical distribution and show that Helmholtz free energy is given by $A = Nk_BT \ln\left(\dfrac{\hbar\omega}{k_BT}\right)$. 2+2

   (ii) Find an expression for the entropy of the system. 2

   (iii) Show that the internal energy of the system is $U = Nk_BT$. 2

4. (a) Plot and compare Fermi-Dirac, Bose-Einstein, and Maxwell-Boltzmann distribution function as a function of energy. 3

   (b) Show that at $T = 0$, the average energy of an electron in a metal is $\frac{3}{5}E_F$ where, E_F denotes the Fermi energy. 4

   (c) What is Gibbs paradox and how is it resolved? 3

5. (a) What is Bose-Einstein statistics? What are the basic postulates used? Derive an expression $n_i = g_i/(e^{\alpha}e^{\beta E_i} - 1)$ for the most-probable distribution of the particles of a system obeying B.E. statistics, hence deduce Planck's blackbody radiation formula. 1+1+3+3

   (b) Consider N non-interacting two level system with energy $\pm\varepsilon$. Show that the maximum entropy is $Nk_B \ln 2$. 2

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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