



WEST BENGAL STATE UNIVERSITY
B.Sc. Programme 6th Semester Examination, 2022

MTMGDSE04T-MATHEMATICS (DSE2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

GROUP-A

Full Marks-10

1. Answer any **five** questions from the following: 2×5 = 10

- (a) Is the set $X = \{(x, y) : x^2 + y^2 \leq 4\}$ is convex? Justify your answer.
- (b) In the following equations find the basic solution with x_3 as the non-basic variable

$$x_1 + 4x_2 - x_3 = 3$$

$$5x_1 + 2x_2 + 3x_3 = 4$$
- (c) Find a basic feasible solution of the equations $x_1 + x_2 + x_3 = 8$, $3x_1 + 2x_2 = 18$
- (d) Find the extreme points, if any, of the set $S = \{(x, y) : 2x + 3y = 6\}$
- (e) Draw the convex hull of the points $(0, 0)$, $(0, 1)$, $(1, 2)$, $(1, 1)$, $(4, 0)$.
- (f) Write down the dual of the following L.P.P.:

$$\text{Maximize } Z = 3x_1 + 5x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 5$$

$$x_1 - x_2 = 7$$

$$x_1, x_2 \geq 0$$

- (g) Determine the position of the point $(-1, 2, 5, 3)$ relative to the hyperplane

$$4x_1 + 6x_2 + x_3 - 3x_4 = 4$$

- (h) Find the number of basic feasible solutions of the following L.P.P.:

$$\text{Maximize } Z = 2x_1 + 3x_2$$

$$\text{Subject to } x_1 + x_2 \geq 2$$

$$x_1 - x_2 \leq 1$$

$$x_1, x_2 \geq 0$$

- (i) What is the criterion for no feasible solution in two-phase method?

GROUP-B**Full Marks-40****Answer any five questions from the following****8×5 = 40**

2. (a) Solve the following L.P.P using graphical method
- 4

$$\begin{aligned}
 &\text{Maximize} && Z = 2x_1 + x_2 \\
 &\text{Subject to} && 4x_1 + 3x_2 \leq 12 \\
 &&& 4x_1 + x_2 \leq 8 \\
 &&& 4x_1 - x_2 \leq 8 \\
 &&& x_1, x_2 \geq 0
 \end{aligned}$$

- (b) Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12 p./gm. Food Y contains 8 units of vitamin A and 12 units of vitamin B per gram and costs 20 p./gm. The daily requirements of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P. to minimize the cost. 4

3. (a) Use Simplex method to solve the L.P.P.
- 4

$$\begin{aligned}
 &\text{Maximize} && Z = x_1 + 2x_2 + 4x_3 \\
 &\text{Subject to} && 3x_1 + 5x_2 + 2x_3 \leq 6 \\
 &&& 4x_1 + 4x_3 \leq 7 \\
 &&& 2x_1 + 4x_2 - x_3 \leq 10 \\
 &&& x_1, x_2, x_3 \geq 0
 \end{aligned}$$

- (b) Show that the vectors $(1, -2, 0), (3, 1, 2), (5, -1, 4)$ form a basis in E^3 . 4

4. (a) Prove that the set of all convex combinations of a finite number of points is a convex set.
- 5

- (b) Find a supporting hyperplane of the convex set
- 3

$$S = \{(x, y) : x + 2y \leq 4, 3x + y \leq 6, x \geq 0, y \geq 0\}$$

5. (a)
- $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 0$
- is a feasible solution of the system of equations
- 5

$$x_1 + 2x_2 + 4x_3 + x_4 = 7$$

$$2x_1 - x_2 + 3x_3 - 2x_4 = 4$$

Reduce the feasible solution to two different basic feasible solutions.

- (b) Prove that a hyperplane is a convex set.
- 3

6. (a) Obtain a basic feasible solution of the following L.P.P. from the feasible solution
- $(2, 3, 1)$
- 4

$$\begin{aligned}
 &\text{Maximize} && Z = x_1 + 2x_2 + 4x_3 \\
 &\text{Subject to} && 2x_1 + x_2 + 4x_3 = 11 \\
 &&& 3x_1 + x_2 + 5x_3 = 14 \\
 &&& x_1, x_2, x_3 \geq 0
 \end{aligned}$$

(b) Prove that the intersection of two convex sets is also a convex set. 4

7. (a) Solve by Charnes Big M-method the following L.P.P. 6

$$\begin{array}{ll}\text{Maximize} & Z = 4x_1 + x_2 \\ \text{Subject to} & 3x_1 + x_2 = 3 \\ & 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0\end{array}$$

(b) Discuss whether the set of points $(0, 0), (0, 1), (1, 0), (1, 1)$ on the xy -plane is a convex set or not. 2

8. (a) Prove that dual of a dual is a primal. 4

(b) Obtain the dual problem of the following L.P.P. 4

$$\begin{array}{ll}\text{Maximize} & Z = -x_1 + 3x_2 \\ \text{Subject to} & 2x_1 + x_2 \leq 1 \\ & 3x_1 + 4x_2 \leq 5 \\ & x_1 + 6x_2 \leq 9 \\ & x_1, x_2, x_3 \geq 0\end{array}$$

9. (a) Find the points which generate the convex polyhedron 3

$$S = \{(x_1, x_2) \in E^2 : x_1 + 2x_2 \leq 4, x_1 - 2x_2 \leq 2, x_1 \geq 0, x_2 \geq 0\}$$

(b) Use two-phase method to solve the following L.P.P. 5

$$\begin{array}{ll}\text{Maximize} & Z = 3x_1 + 5x_2 \\ \text{Subject to} & x_1 + 2x_2 \geq 8 \\ & 3x_1 + 2x_2 \geq 12 \\ & 5x_1 + 6x_2 \leq 60 \\ & x_1, x_2 \geq 0\end{array}$$

N.B. : Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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