16/12/19

1st

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2019

MTMACOR07T-MATHEMATICS (CC7)

NUMERICAL METHODS

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *four* from the rest

1. Answer any ***four*** questions from the following: $2\times 4=8$

(a) Find the numbers of significant figures in the approximate number 0.4785, given its relative error as 0.2×10^{-2}.

(b) Prove that E^{-1} is a linear operator.

(c) Prove that $f(x_0, x_1, x_2)=1$ if $f(x)=x^2$, where x_0, x_1, x_2 are distinct arguments.

(d) Find the dominant eigen value (if exists) of the matrix.

$$A=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(e) Find a polynomial $f(x)$ for which $f(0)=2,\ f(1)=1,\ f(2)=0$.

(f) What are the advantages and disadvantages of Newton-Raphson method?

(g) Apply Trapezoidal rule to evaluate $\int_0^1 e^{-x^2}dx$ with 4 divisions.

2. (a) Prove for equally spaced interpolating points $x_i = x_0 + ih\ \ (h>0,\ i=0, 1, \ldots\ldots, n)$ 3

$$\Delta^k y_0 = \sum_{i=0}^{k}(-1)^i \binom{k}{i} y_{k-i}.$$

(b) Prove that $f(x_0, x_1, \ldots, x_n) = \sum_{i=0}^{n} \dfrac{f(x_i)}{\prod\limits_{\substack{j=0 \\ j\neq i}}^{n}(x_i - x_j)}$, where $f(x_0, x_1, \ldots, x_n)$ represents the nth order divided difference of arguments $x_0, x_1, \ldots, x_n$. 5

3. (a) Find an expression for the maximum error in linear interpolation of a twice differentiable function $f(x)$ for the arguments x_0 and x_1. 3

(b) Derive Lagrange interpolation formula for given n interpolating points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$. 5

4. Derive Newton-Cotes' closed type formula for evaluating an integral of the form $\int_a^b f(x)\,dx$ where a, b are finite. Hence or otherwise obtain Boole's formula for numerical integration. 4+4

5. Use Picard's method to approximate $y(x)$ (up to 4th approximation) for the initial value problem $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$. Compare it with the infinite series expansion of the exact solution. 5+3

6. (a) Use Runge-Kutta method of order four to calculate $y(1\cdot 1)$ taking $h = 0.05$ for the equation $\frac{dy}{dx} = xy^{\frac{1}{3}}$, $y(1) = 1$ correct upto four decimal places. 5

(b) Use Euler's method to find the solution of the differential equation $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ for $x = 0.3$ taking $h = 0.1$. 3

7. (a) Describe Bisection method for computing a simple real root of $f(x) = 0$. Give a geometrical interpretation of the method and also the error estimate. 4+1+1

(b) Discuss the advantages and disadvantages of Bisection method. 2

8. (a) Describe the LU decomposition method for finding the inverse of a matrix. 5

(b) From the following table, find the area bounded by the curve and the x-axis from $x = 7.47$ to $x = 7.52$ by trapezoidal rule: 3

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

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