WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

MTMACOR02T-MATHEMATICS (CC2)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Solve the equation $x^5+x^4+x^3+x^2+1=0$.

(b) Express $(\sqrt{3}+i)$ in polar form and hence find $(\sqrt{3}+i)^{120}$.

(c) If each of the four positive real numbers a, b, c, d is greater than 1, show that $8(abcd+1) > (a+1)(b+1)(c+1)(d+1)$.

(d) If α is a root of the cubic equation $x^3-3x+1=0$, then find the other two roots are α^2-2 and $2-\alpha-\alpha^2$.

(e) Using Descarte's rule of sign, find the nature of the roots of the equation $x^4+16x^2+7x-11=0$.

(f) Let a, b be two nonzero integers and c be an integer. If $a\mid c$, $b\mid c$ and $\gcd(a, b)=1$, show that $ab\mid c$ (the symbol $m\mid n$ means 'm divides n').

(g) If 2 and 3 are the eigenvalues of a real square matrix A of order 2, find by applying Cayley-Hamilton theorem the inverse of A in terms of itself.

(h) For a finite set S, if $f:S\to S$ be injective, then show that f is bijective.

2. (a) Use De Moivre's theorem to show that $\sin^4\theta\cos^4\theta=\frac{1}{2^7}(\cos 8\theta-4\cos 4\theta+3)$. 3

(b) Solve the equation $2x^4-5x^3-15x^2+10x+8=0$, when it is given that the roots of the equation are in geometric progression. 3

(c) Prove that the roots of the equation $\frac{1}{x-1}+\frac{2}{x-2}+\frac{3}{x-3}=x$ are all real. 2

3. (a) Solve by Ferrari's method: $9x^4+12x^3+9x^2-2x-8=0$. 5

(b) If $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ are $2n$ real numbers, then show that 3

$$(a_1b_1+a_2b_2+\ldots+a_nb_n)^2 \le (a_1^2+a_2^2+\ldots+a_n^2)(b_1^2+b_2^2+\ldots+b_n^2).$$

4. (a) Let x, $y\in\mathbb{Z}$ (the set of all integers) with $y\neq 0$. Then show that there exist unique integers q and r such that $x=qy+r$, $0\le r<|y|$. 5

(b) Define inverse of a relation on a nonempty set. Prove that a relation ρ on a nonempty set S is symmetric if and only if $\rho=\rho^{-1}$ where ρ^{-1} stands for the inverse of ρ. 3

5. (a) Show that there is a mapping $\phi: \mathbb{Z} \longrightarrow \mathbb{Z}$ which is injective but not surjective, $\mathbb{Z}$ being the set of all integers. 2

(b) Let $f: A \to B$, $g: B \to C$, $h: B \to C$ be three mappings such that f is surjective and $g \circ f = h \circ f$. Prove that $g = h$. 3

(c) Prove that the set of all integers $\mathbb{Z}$ and the set of all natural numbers $\mathbb{N}$ are of same cardinality. 3

6. (a) Let a and $b(\geq 1)$ be integers. Prove that there exist unique integers q and r such that $a = bq + r$ with $0 \leq r < b$. 3

(b) Using mathematical induction, find the least positive integer n_0 such that $n^2 < n!$ for all positive integers $n \geq n_0$. 3

(c) Prove that an integer $p > 1$ is a prime number if and only if p divides ab implies, either p divides a or p divides b, where a and b are any two integers. 2

7. (a) Use the notion of congruence relation between the integers, to prove that 41 divides $2^{20} - 1$. 3

(b) Let a, b, c be integers and m be a positive integer. Prove that $ab \equiv ac \pmod{m}$ if and only if $b \equiv c\left(\bmod \dfrac{m}{\gcd(a, m)}\right)$. 3

(c) Show that $\phi(5n) = 5\phi(n)$ if 5 divides n, where n is a positive integer and ϕ denotes the Euler phi function. 2

8. (a) Prove that $\gcd(n, n+1) = 1$ for any $n \in N$. Find integers x and y such that $nx + (n+1)y = 1$. 5

(b) For a positive integer a, find the integral value of b for which the following system of equations will have infinitely many solutions: 3

$$x + y + z = 1$$
$$x + 2y - z = b$$
$$5x + 7y + a^2 z = 5b^2$$

9. (a) Applying elementary row operations, find the inverse of the matrix 3

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 4 \\ 1 & -2 & 1 \end{bmatrix}$$

(b) Find the rank of the matrix $A = \begin{bmatrix} a & -1 & -1 \\ -1 & a & -1 \\ -1 & -1 & a \\ 1 & 1 & 1 \end{bmatrix}$ for different real values of a. 3

(c) If λ is an eigenvalue of a real orthogonal matrix A, prove that $\frac{1}{\lambda}$ is also an eigenvalue of A. 2

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