लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 3rd Semester Examination, 2019

MTMHGEC03T/MTMGCOR03T-MATHEMATICS (GE3/DSC3)

REAL ANALYSIS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Express real line in terms of a set.

(b) Justify that every real number is a cluster point of $\mathbb{Q}$, where $\mathbb{Q}$ is the set of rational numbers.

(c) Show that every bounded sequence is not convergent.

(d) Show that pointwise convergence may not imply uniform convergence.

(e) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n}{2^n}$.

(f) Find the limit function $f(x)$ of the sequence $\{f_n\}$ where

$$f_n(x)=\frac{nx}{1+nx},\quad x\geq 0$$

(g) Use Weierstrass' M-test to show that the series

$$\sum_{n=1}^{\infty}\frac{n^5+1}{n^7+3}\left(\frac{x}{2}\right)^n$$

converges uniformly in $[-2, 2]$

(h) Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{n^n x^n}{n!}$$

2. (a) State and prove Archimedean property of $\mathbb{R}$. 4

(b) Let A be a non empty bounded above subset of $\mathbb{R}$. Let 4

$$-A=\{-x:\ x\in A\}.$$

Show that $-A$ is a non empty bounded below subset of $\mathbb{R}$ and $\inf(-A)=-\sup A$.

3. (a) Show that $\mathbb{N}$ is unbounded above. 3

(b) Prove that the open interval $(0, 1)$ is uncountable. 5

4. (a) Does every infinite subset of real numbers have at least one cluster point? Justify your answer. 2

(b) Does every bounded subset of real numbers have at least one cluster point? Justify your answer. 2

(c) Find the cluster points of the set 4

$$S=\left\{ (-1)^{n+1}\frac{n+2}{n+1} : n\in\mathbb{N} \right\}$$

5. (a) Show that the sequence $\left\{\left(1+\frac{1}{n}\right)^{n+1}\right\}$ is a monotone decreasing sequence and find its limit. 4

(b) Show that $\lim\limits_{n\to\infty} x_n = 1$, where 4

$$x_n=\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}},\ \forall n\in\mathbb{N}$$

6. (a) Test the convergence of the series 4

$$\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+1}x^n, \text{ where } x\neq 1.$$

(b) Test the convergence of the series 4

$$\sum_{n=1}^{\infty}(-1)^{n-1}\cdot\frac{2n+1}{n(n+1)}$$

7. (a) State and prove Cauchy's first theorem. 4

(b) Find the limit function $f(x)$ of the sequence $\{f_n\}$ where for all $n\in\mathbb{N}$, 4

$$f_n(x)=\frac{nx}{1+n^2x^2},\ 0\leq x\leq 1$$

Also show that the sequence $\{f_n(x)\}$ is not uniformly convergent on $[0,1]$.

8. (a) Use Cauchy's general principle of convergence to show that the sequence $\left\{\frac{n}{n+1}\right\}$ is convergent. 4

(b) Find the sum function of the series 2+2

$$x^4+\frac{x^4}{1+x^4}+\frac{x^4}{(1+x^4)^2}+\frac{x^4}{(1+x^4)^3}+\cdots$$

where $0\leq x\leq 1$. Hence state with reason whether the series is uniformly convergent on $[0,1]$.

9. (a) Find the radius of convergence of the power series 4

$$\sum_{n=0}^{\infty}[3+(-1)^n]^n x^n$$

(b) Assuming the power series expansion 4

$$\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n \text{ for } |x|<1,$$

show that $\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\cdots;\quad |x|<1.$

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