WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

MTMADSE02T-MATHEMATICS (DSE1/2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Define Linear Congruence.

(b) State Wilson's Theorem.

(c) Find $\phi(260)$.

(d) If m is an odd prime and a be an integers such that $(a, m)=1$, then prove that $\left(\frac{a}{m}\right) \equiv a^{(m-1)/2}$ (mod m). Where $\left(\frac{a}{m}\right)$ denotes Legendre symbol.

(e) State Fermat's last theorem.

(f) Find all solutions of the Diophantine equation $3x+2y=6$.

(g) Determine which of the following integers are primes:
(i) 287, (ii) 271

(h) Write down Mobius Inversion Formula.

(i) Write down the statement of Chinese Remainder Theorem.

(j) Without performing the long divisions, determine whether the integer 761215122 is divisible by 9 or 11 or 3.

2. (a) A fruit seller orders mangoes and oranges for Rs. 1,000. If one basket of mangoes costs Rs. 20, and one basket of oranges costs Rs. 172, how many baskets of each type does he order? 5+3

(b) Which of the following Diophantine equations cannot be solved?
(i) $6x+4y=91$ (ii) $621x+736y=46$ (iii) $158x-57y=7$

3. (a) Define prime counting function. 3+2+3

(b) State the theorem of prime numbers.

(c) Why is Goldbach's conjecture important?

4. (a) Find all solution of $7x\equiv 4$ (mod 18). 5+3

(b) Find the inverse of 7 modulo 10.

5. (a) A certain integer between 1 and 1000 leaves the remainder 1, 2, 6 when divided by 9, 11, 13 respectively. Find the integer. 4+4

(b) Prove Fermat's Little theorem.

6. (a) If p and q are any pair of distinct prime numbers prove that $\phi(pq)=(p-1)(q-1)$, when ϕ is Euler's phi function. 3+5
   (b) Find the remainder when 17! is divided by 19.

7. (a) If a, n be integers such that $n>0$ and $\gcd(a, n)=1$, then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$. 4+4
   (b) Prove that $\phi(5n)=5\phi(n)$ if and only if 5 divides n.

8. (a) If n be a positive integer such that $\gcd(n, 9)=1$, prove that 9 divides $n^{18}-1$. 3+3+2
   (b) Let $n>2$ be an integer, show that $\phi(n)$ is even.
   (c) Define Euler's phi function.

9. (a) Find four primitive roots of 25. 6+2
   (b) If a is a primitive root of p, then prove that $a+p$ is also its primitive root where a is an odd prime.

10. (a) If m is an odd prime > 2, prove that the product of primitive roots of m is congruent to $1 \pmod{m}$. 4+2+2
    (b) The prime $m=71$ has 7 as a primitive root. Find all primitive roots of 71 and also find a primitive root of m^2.

11. (a) Solve $x^2+7x+10 \equiv 0 \pmod{11}$. 4+4
    (b) Prove Euler's criterion.

12. (a) Define Legendre symbol $\left(\frac{a}{p}\right)$. 2+3+3
    (b) Prove that, for an odd prime p

    (i) $\left(\frac{a^2}{p}\right)=1$ and (ii) $\left(\frac{a}{p}\right)\left(\frac{b}{p}\right)=\left(\frac{ab}{p}\right)$

13. (a) Let m be an odd prime and $(a, m)=1$. Establish that the quadratic congruence $ax^2+bx+c \equiv 0 \pmod{m}$ is solvable, if and only if b^2-4ac is either zero or a quadratic residue of m. 5+3
    (b) Prove that there exist infinitely many primes of the form $4n+1$.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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