WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

MTMADSE06T-MATHEMATICS (DSE3/4)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

   (a) Define astatic equilibrium and astatic centre.

   (b) Define coefficient of friction and the angle of friction.

   (c) State the principle of virtual work for a system of coplanar forces acting on a rigid body.

   (d) Find the centre of gravity of a uniform rectangular lamina with sides of length a and b.

   (e) Weights proportional to 1, 4, 9 and 16 are placed in a straight line so that the distance between them are equal; find the position of their centre of gravity.

   (f) Describe stable and unstable equilibrium and the position of the centre of gravity in each case.

   (g) Write down the equations of motion of a particle projected with a velocity u making an angle α with the horizon in a medium offering resistance proportional to the velocity.

   (h) Define equimomental bodies. State the necessary and sufficient condition for two systems to be equimomental.

   (i) If a rigid body rotates about a space-fixed axis, θ be the angular velocity of the body about the axis at any instant and Mk^2 the moment of inertia of the body about the axis, then prove that the kinetic energy of the body at that instant is $\frac{1}{2}Mk^2\dot{\theta}^2$.

2. (a) Forces P, Q, R act along the x-axis, y-axis and the straight line $x\cos\alpha + y\sin\alpha = p$. Find the magnitude of the resultant and the equation of the line of action. 4

   (b) A solid homogeneous hemisphere rests on a rough horizontal plane whose coefficient of friction is μ' and against a rough vertical wall with coefficient of friction is μ. Show that the least angle that the base of the hemisphere can make with the vertical is $\cos^{-1}\left(\frac{8\mu'}{3}\frac{1+\mu}{1+\mu\mu'}\right)$. 4

3. (a) Three forces P, Q, R act along the three straight lines $x=0$, $y-z=a$; $y=0$, $z-x=a$; $z=0$, $x-y=a$ respectively. Show that P, Q, R cannot reduce to a couple. 4

   (b) The density at any point of a circular lamina varies as the n-th power of the distance from a point O on the circumference. Show that the centre of gravity of the lamina divides the diameter through O in the ratio $n+2:2$. 4

4. (a) State the principle of virtual work for any system of coplanar forces acting on a rigid body. 2

   (b) A regular pentagon *ABCDE* is formed of five uniform heavy rods, each of weight *W* and freely joined at their extremities. It is freely suspended from *A* and is maintained in its regular pentagon form by light rod joining *B* and *E*. prove that the stress in this rod is $W\cot(18°)$. 6

5. A solid hemisphere rests on a plane inclined to the horizon at an angle $\alpha < \sin^{-1}\frac{3}{8}$ and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable. 8

6. (a) If the axes *Ox*, *Oy* revolve with constant angular velocity *w* and the components of velocities of the point (*x*, *y*) are px and py, where $p=\frac{a^2-b^2}{a^2+b^2}w$, prove that the point describes relatively to the axes an ellipse. Find also its periodic time. 3+1

   (b) A body describing an ellipse of eccentricity *e* under the action of a force directed to focus when at the nearer apse, the centre of force is transferred to the other focus. Prove that eccentricity of the new orbit is $e\frac{(3+e)}{(1-e)}$. 4

7. A particle is projected at right angles to the line joining it to a centre of force, attracting according to the law of inverse square of the distance, with a velocity $\frac{\sqrt{3}V}{2}$, where *V* is the velocity from infinity. Find the eccentricity of the orbit described and show that the periodic time is $2\pi T$, *T* being the time taken to describe the major axis of the orbit with velocity *V*. 8

8. A particle of given mass be moving in a medium whose resistance varies as the velocity of the particle. Show that the equation of the trajectory can, by a proper choice of axes be put into the form $y+ax=b\log x$. 8

9. (a) Using the necessary condition for a given straight line to be a principal axis at some point of its length, prove the following:

   (i) through each point of a plane lamina there exists a pair of principal axis of the lamina, 2

   (ii) if an axis passes through the centre of gravity of a body and is a principal axis at any point of its length, then it is a principal axis at all points of its length. 2

   (b) Show that for a rigid body the motion of centre of inertia is independent of the motion relative to the centre of inertia. 4

10. (a) State the principle of conservation of momentum both for finite and impulsive forces. State also the principle of conservation of energy. 4

    (b) A solid homogeneous cone of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4+\tan^2\alpha)$. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

———×———