WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2019

MTMACOR04T-MATHEMATICS (CC4)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

1. Answer any ***five*** questions from the following: $2\times 5=10$

(a) Find a particular integral of the differential equation $(D^2+1)y=\cos 2x$.

(b) Convert the following equation to a linear equation with constant coefficient

$$(x^3D^3+2x^2D^2+1)y=10(x+\tfrac{1}{x})$$

(c) Find the regular singular points, if any, of the differential equation

$$(x-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0$$

(d) Write down the normal linear system for the following differential equation of order 3:

$$\frac{d^3x}{dt^3}+3\frac{d^2x}{dt^2}-4x=te^{-t}$$

(e) Find a fundamental matrix for the linear system $\dot{x}(t)=Ax(t)$

where $A=\begin{pmatrix}-2 & 3\\ 3 & -2\end{pmatrix}$, $\quad x(t)=\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix}$

(f) If $\vec{a}=\hat{i}-2\hat{j}-3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$, $\vec{c}=\hat{i}+3\hat{j}-2\hat{k}$, find $|\vec{a}\times(\vec{b}\times\vec{c})|$.

(g) Find $\operatorname{grad}\phi$ where $\phi=3x^2y-y^3z^2$ at the point $(1,-2,1)$.

(h) Prove that the three vectors $\alpha\times(\beta\times\gamma)$, $\beta\times(\gamma\times\alpha)$, $\gamma\times(\alpha\times\beta)$ are coplanar.

Answer any *five* questions (from Question 2 to Question 9) $8\times 5=40$

2. (a) Solve: $\dfrac{d^2y}{dx^2}-6\dfrac{dy}{dx}+9y=4x^2$ — 4

(b) Solve: $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+2y=e^{e^x}$ — 4

3. (a) Solve, by the method of undetermined coefficient: — 4

$$(D^2+4)y=\sin 2x$$

(b) Solve: $x^2\dfrac{d^2y}{dx^2}-4x\dfrac{dy}{dx}+6y=\log(x^2)$ — 4

4. (a) Solve: $\frac{d^2x}{dt^2}-2\frac{dy}{dt}-x=e^t\cos t$; $\frac{d^2y}{dt^2}+2\frac{dx}{dt}-y=e^t\sin t$ 4

(b) Obtain the power series solution of $y''+xy'+x^2y=0$ about $x=0$. 4

5. (a) Solve by the method of variation of parameters 4

$$\frac{d^2y}{dx^2}+a^2y=\cos ax$$

(b) Solve: $\frac{d^2y}{dx^2}-y=x^2\cos x$ 4

6. (a) Solve the linear homogeneous system $\dot{x}(t)=Ax(t)$ 6

where $A=\begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, $x(t)=\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$

(b) Show that the function $f(x, y)=xy^2$ satisfy Lipschitz condition in $|x|\le 1, |y|\le 2$. 2

7. (a) Determine the nature of the critical point (0, 0) of the autonomous system 3

$\dot{x}=x+3y$; $\dot{y}=3x+y$

(b) Show that origin is a regular singular point of the differential equation $2x^2y''+xy'-(x+1)y=0$ and find its power series solution at $x=0$. 5

8. (a) A particle moves so that its position vector is given by $\boldsymbol{r}=\cos\omega t\boldsymbol{i}+\sin\omega t\boldsymbol{j}$, where ω is a constant. Show that 2+2+1

(i) The velocity **v** is perpendicular to **r**.

(ii) The acceleration **a** is directed towards the origin and has magnitude proportional to the distance from the origin.

(iii) $\mathbf{r}\times\mathbf{v}$ is a constant vector.

(b) Find *a*, *b*, *c* such that $\vec{F}=(6xy+az^3)\hat{i}+(bx^2-z)\hat{j}+(3xz^2+cy)\hat{k}$ is irrotational. 3

9. (a) Prove that the necessary and sufficient condition that a vector function $\boldsymbol{f}(t)$ has a constant direction is $\boldsymbol{f}\times\frac{d\boldsymbol{f}}{dt}=\boldsymbol{0}$. 3

(b) (i) If $\boldsymbol{\alpha}=t^2\boldsymbol{i}-t\boldsymbol{j}+(2t+1)\boldsymbol{k}$ and $\boldsymbol{\beta}=(2t-3)\boldsymbol{i}+\boldsymbol{j}-t\boldsymbol{k}$, where ***i***, ***j***, ***k*** have their usual meaning, then find $\frac{d}{dt}(\boldsymbol{\alpha}\times\frac{d\boldsymbol{\beta}}{dt})$ at $t=2$. 3+2

(ii) If $\boldsymbol{r}(t)=5t^2\boldsymbol{i}+t\boldsymbol{j}-t^3\boldsymbol{k}$, then find the values of $\int_1^2(\boldsymbol{r}\times\frac{d^2\boldsymbol{r}}{dt^2})\,dt$.

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